Interaction of a Sphere with a Suspension of Other Spheres

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The determination of the fluid dynamic interaction force exerted on a solid rigid particle by a surrounding fluid is at the core of many theoretical and experimental research studies. The relevance of this problem is twofold, with, on the one hand, the objective of exploring and understanding basic fluid dynamic knowledge and, on the other hand, for chemical and process engineering, with the objective of obtaining reliable information for the correct design and operation of solid-fluid multiphase units such as settlers, crystallizers, and so on.

The state of the art is well known; the simplest possible case is that of a single solid sphere in a infinite expanse of fluid for which, when inertial effects can be ignored, the terminal settling velocity is

$$u_t = \frac{d^2(\rho_p - \rho)g}{18\mu} \tag{1}$$

Theory is of limited help when inertia becomes important and estimation of the fluid-particle interaction force has been obtained from direct measurements, which are summarized with empirical relationships for the drag coefficient (for example, Dallavalle, 1948).

Further complications are encountered when more than one sphere is present. Both the buoyancy and the drag force on a particle are affected by the presence of other particles, which have the effect of increasing the interaction force as the solid concentration increases. Theoretical analyses are bound to be limited to systems where inertia can be ignored and, in the case of identical spheres, the result of Batchelor (1972), expressed in terms of steady-state settling velocity u_i is acknowledged as being the most trustworthy

$$u_i = u_{t,i}(1 + S_{ii}\phi_i) \tag{2}$$

The above equation indicates a linear relationship between particle settling velocity and particle concentration ϕ_i with the value of the coefficient S_{ii} equal to -6.55. Batchelor's results are only valid for dilute conditions (a typical upper limit is assumed to be 5% in solid volume concentration). Outside that limit, expressions for the interaction force have been suggested using experimental suspension behavior as a source of information summarized, for example, by the Richardson and Zaki (1954) velocity-concentration relationship

$$u_i = u_{t,i}(1 - \phi_i)^n \tag{3}$$

where n is an empirical parameter function of the prevailing flow regime.

The most general case encountered is that of suspensions made up of spheres differing both in size and density: in this case theoretical analyses are obviously also limited to negligible inertia and dilute concentrations; again, Batchelor (1982) presented an expression for the settling velocity of each particle species in the form

$$u_i = u_{t,i}\left(1 + \sum_j S_{ij}\phi_i\right) \tag{4}$$

where the coefficient S_{ij} is a function of two parameters, namely

$$\lambda = \frac{d_j}{d_i} \tag{5}$$

and

$$\gamma = \frac{\rho_j - \rho}{\rho_i - \rho} \tag{6}$$

The dependency of the settling velocity on the solid concentrations is again linear and Batchelor and Wen (1982) calculated values of S_{ij} for selected pairs of λ and γ. For negligible interparticle forces and large Peclet numbers, these values are reported in Table 1. In the same Table, S_{ij} values calculated by Reed and Anderson (1980), in a somewhat less rigorous manner, are shown for those λ-γ pairs not explicitly

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Table 1. Coefficient $S_{i,j}$ Calculated by: Batchelor and Wen (1982) vs. Reed and Anderson (1980)*

		Numerical Values, γ											
		−2	**−1**	**−0.5**	**0**	**0.25***	**0.5***	**0.6**	**1**	**1.5**	**2***	**2.25**	**4***
λ	**0.25**	−1.96	−2.00	−2.20	−2.56	−2.31	−2.76	−3.31	−3.83	−4.73	−5.38	−6.90	−8.88
	0.5	−2.51	−2.27	−2.28	−2.53	−2.83	−3.48	−3.41	−4.29	−6.77	−7.78	N.A.	−12.6
	1	−2.26	−2.39	−2.45	−2.52	−3.02	−4.18	−2.60	−6.55	−2.71	−11.2	−2.81	−20.5
	2	3.18	−0.34	−1.89	−2.44	−4.11	−6.70	−9.85	−9.81	−11.16	−22.2	−13.71	−43.0
	4	26.63	10.05	2.03	−2.66	−8.0	−15	−19.55	−24.32	−32.71	−56.8	N.A.	−113

listed in Batchelor and Wen's article. Reed and Anderson (1980) observed in their work that a halving of γ roughly counteracts a doubling of λ on the numerical value of the coefficient S_{ij}, and vice versa.

It is easy to understand the importance of theoretical studies, but in spite of this, experiments aimed at supporting Batchelor's theoretical conclusion are scarce. Ham and Homsy (1988) studied the settling velocity of a single sphere in a suspension of identical spheres ($\lambda = \gamma = 1$): for dilute conditions the experimental S_{ii} coefficient was in the region of -4. Davis and Birdsell (1988) studied the settling velocity of bidisperse and tridisperse sphere mixtures in dilute suspensions: they investigated the behavior of glass ballotini of 136, 186, and 261 μm in size (therefore, γ was always equal to 1 and λ varied in the range 0.52−1.92) and found an excellent agreement with Batchelor's theoretical predictions. No other investigations appear to have been carried out for systems of noncolloidal particles; the objective of this article is to fill that gap.

Experimental Studies

Measuring the settling velocity of solid particles in dilute suspensions poses practical problems, such as interface spreading and velocity fluctuations, which, in some cases, can affect the reliability of the conclusions (Davis, 1996). In order to overcome this difficulty, a limiting situation was investigated, which was much simpler experimentally, however, at the same time, still allowed for a confident comparison with theoretical predictions: the settling velocity of a single particle, indicated by p, in a dilute suspension, indicated by s.

For this limiting situation, Eq. 4 reduces to

$$u_p = u_{t,p}(1 + S_{p,s}\phi_s) \tag{7}$$

with $S_{p,s}$ a function of

$$\lambda = \frac{d_s}{d_p} \quad \text{and} \quad \gamma = \frac{\rho_s - \rho}{\rho_p - \rho} \tag{8}$$

The experimental procedure was fairly simple. A certain amount of suspension particles and fluid (silicon oil for all the runs, with a density of 970 kg/m^3 and viscosity of 1.2 Pa·s at 20°C) were charged into a sedimenting column, a transparent cylindrical vessel 500 mm in height and 107 mm in diameter. Turning the column upside down and rotating it for a few minutes yielded a good visually homogeneous suspension. At this point, the top cover of the column was removed and a single particle was introduced in the middle; the single particle was chosen so that it would settle faster than the suspension particles, and this settling velocity was measured with a stopwatch with marks on the column wall placed every 10 mm. The measurement took place after the test particle had traveled some 100 mm, in order to be sure that terminal conditions were reached. The procedure was repeated at least five times and on average ten times for each condition. The temperature of the fluid was monitored between each run and kept within ±0.5°C of the initial value by replacing some of the oil in the column with fresh oil. The solid particles were all spherical, and their characteristics are reported in Table 2. Suspension particle size was determined by measuring 50 individual particles: the average of those measurements is the value indicated in Table 2 (a size spread smaller than 10% was observed in all cases). Test particles were measured individually with a micrometer and accepted only if the size was within ±0.05 mm of the set value. A wide range of λ (0.17−2) and γ (0.04−6.51) were investigated.

Table 2. Physical Properties of the Solid Used

	Test Sphere			Suspension Sphere				
System No.	Material	Density (kg/m^3)	Size (mm)	Material	Density (kg/m^3)	Size (mm)	λ	γ
1	Glass	2,550	6	Plastic	1,280	3	0.5	0.19
2	Glass	2,550	6	Glass	2,550	1.7	0.29	1
3	Glass	2,550	6	Zirconia	3,800	1.2	0.20	1.76
4	Glass	2,550	6	Copper	8,800	1	0.17	4.89
5	Glass	2,550	6	Lead	11,400	1.5	0.25	6.51
6	Glass	2,550	3	Glass	2,550	1.7	0.57	1
7	Lead	11,400	3.5	Plastic	1,280	3	0.85	0.04
8	Lead	11,400	3.5	Glass	2,550	1.7	0.48	0.15
9	Lead	11,400	3.5	Zirconia	3,800	1.2	0.34	0.27
10	Lead	11,400	3.5	Copper	8,800	1	0.29	0.75
11	Lead	11,400	3.5	Lead	11,400	1.5	0.42	1
12	Lead	11,400	3.5	Glass	2,550	6	1.71	0.15
13	Lead	11,400	1.5	Glass	2,550	1.7	1.14	0.15
14	Lead	11,400	1.5	Glass	2,550	3	2	0.15

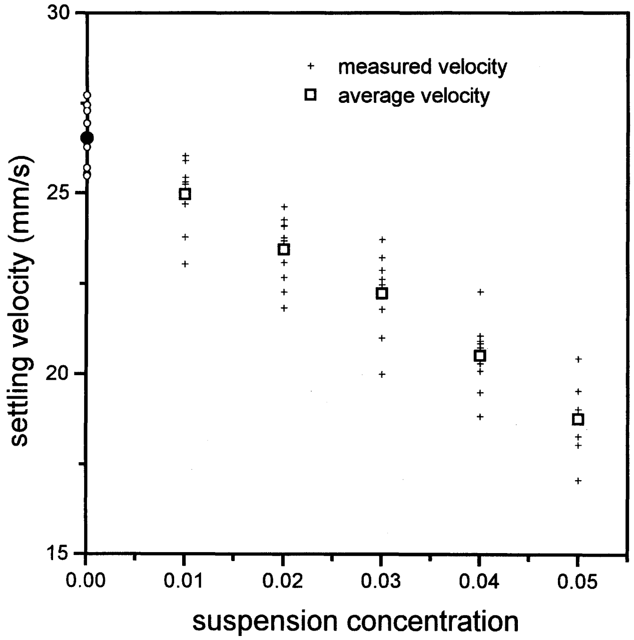

Figure 1. Settling velocity of a 6 mm glass sphere in a 1.2 mm zirconia–silicon oil suspension.

The larger symbols represent calculated average velocities.

The suspensions were all in viscous flow conditions and in the dilute regime with suspension solid concentration up to maximum values of 5% in volume; this dilution allowed the settling particle to be visible to the naked eye in any condition.

Results

The result of an experimental run is illustrated in Figure 1, with the spread of the measurement being a very typical one. Particle settling velocities were measured at different suspension concentrations; the experimental points on the vertical axis (that is, at $\phi_s = 0$) correspond to the settling velocity in pure fluid. For each suspension concentration, an arithmetic average settling velocity was determined (indicated in Figure 1); however, confidence interval in the parameters estimation reported later were calculated by using all the original experimental data. Possible effects of the container wall on the settling velocity have been ignored, as it has been assumed that their magnitude is the same both in the absence and the presence of suspension particles, as suggested by Brenner et al. (1990).

The linear dependency of the settling velocity with solid concentration, as suggested by Batchelor through Eq. 4, was first of all checked. To this end, settling velocities at the various suspension concentrations, as those depicted in Figure 1, were fitted with a straight line (all the results from the error minimization routine are summarized in Table 3, with the numerical parameters calculated with a confidence interval at a 95% level). The extrapolated velocity at zero concentration was compared with the measured single particle settling velocity in pure fluid: this is reported in Figure 2, and the notably good correspondence between the two velocities support Batchelor's conclusions. Secondly, theoretical and experimental $S_{p,s}$ values were also compared. Figure 3 reports this comparison: experimental $S_{p,s}$ were calculated from the fitting minimization routine, whereas theoretical values were deduced from Table 1. In the majority of cases a simple interpolation was necessary to get a good estimate of $S_{p,s}$, as suggested by Batchelor and Wen (1982) themselves. In analogy with the work of Davis and Gecol (1994), who carried out this exercise for the case of γ equal 1, third-order polynomials were used to fit theoretical data, alternatively along the columns and the rows of the Table 1. For very few of the systems (systems No. 4 and 5), an extrapolation of data in Table 1 had to be carried out and this was done following the suggestion given by Reed and Anderson (1982) reported earlier.

With the exception of only two cases (No. 10 and 11), all the systems investigated behave accordingly with theoretical predictions. Therefore, Batchelor's theory, based on pair interaction, results quite accurately in the dilute regime and accounts fully for the fluid dynamic solid-fluid interactions. It is worthy to stress, once again, the wide range of system characteristics investigated here, compared to previous studies, especially as far as the density ratio γ is concerned.

Table 3. Least-Squares Linear Regression Results

System No.	Extrapolated Velocity at Zero Solid Conc. (mm/s)	Exp. Terminal Settling Velocity (mm/s)	Coeff. $S_{p,s}$ (Exp.)	Coeff. $S_{p,s}$ (Theor.)
1	24.5 ± 0.8	24.6	-2.70 ± 0.41	-2.8
2	27.6 ± 0.8	27.5	3.69 ± 0.60	-3.9
3	25.9 ± 0.8	26.2	-5.77 ± 0.71	-5
4	21.9 ± 0.6	24.8	-9.96 ± 0.69	-9.3
5	23.3 ± 0.8	23.5	-11.40 ± 1.12	-10.8
6	6.1 ± 0.2	6.0	-3.69 ± 0.51	-4.5
7	65.2 ± 1.9	66.0	-2.80 ± 0.50	-2.5
8	66.5 ± 2.0	66.8	-2.81 ± 0.55	-2.7
9	65.7 ± 1.9	66.2	3.28 ± 0.80	-3
10	71.7 ± 2.2	66.3	-6.77 ± 1.02	-3.5
11	62.5 ± 1.8	62.4	-7.13 ± 1.31	-4.2
12	62.9 ± 0.4	63.1	-3.39 ± 0.68	-3.8
13	13.3 ± 0.4	13.1	-2.83 ± 0.43	-2.5
14	13.1 ± 0.7	13.0	-3.68 ± 0.81	-3.8

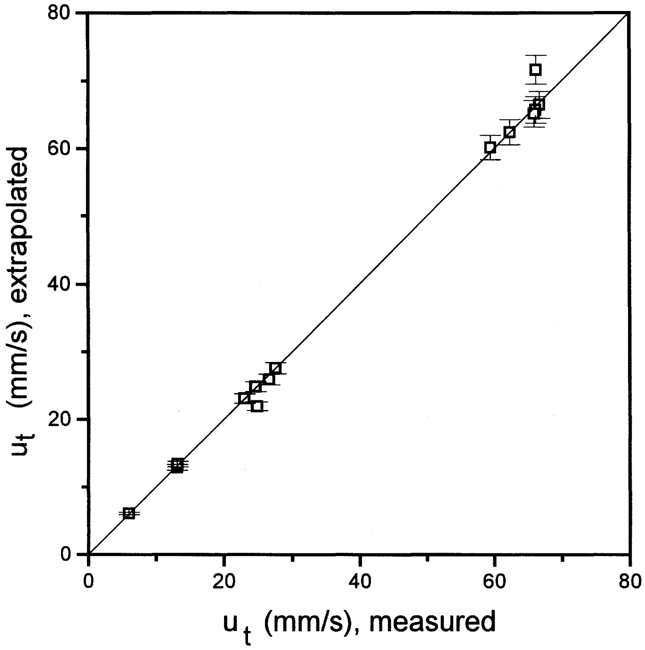

Figure 2. Measured and extrapolated single particle terminal settling velocity.

The error bars have been estimated for a confidence level of 95%.

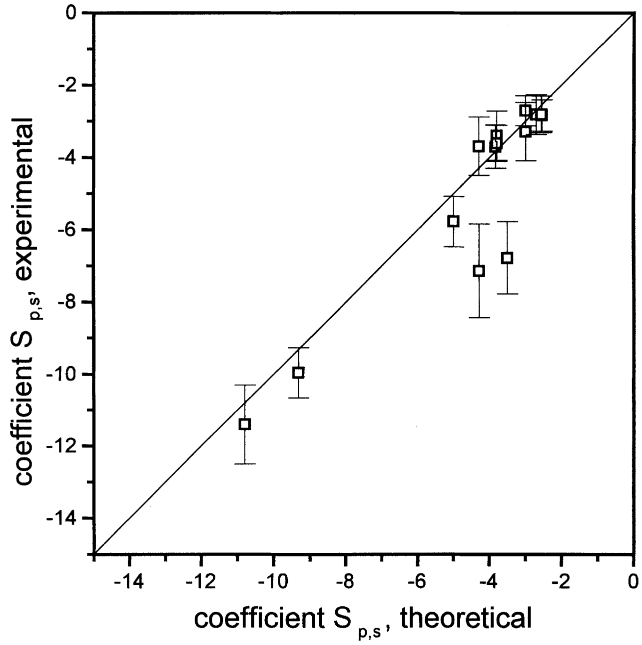

Figure 3. Comparison between measured and theoretical $S_{p,s}$ coefficient.

The error bars have been estimated for a confidence level of 95%.

Pseudo-Fluid Approach

In the previous section the reliability of theoretical predictions has been checked against a substantial amount of experimental evidence: the settling velocity of a sphere in a suspension of others can actually be predicted with satisfactory approximation once the system characteristics are defined. Unfortunately, it can be easily pointed out that such predictions are only valid for conditions (negligible inertia and dilute solid concentration) that are not always encountered in practice. A great deal of effort is being devoted to the widening of the limit of the validity of theoretical works; however, in the meantime, some methodology is needed in order to have a tool which, even if not as rigorous as a fully theoretical approach, would still provide some reliable guesses for practical cases.

A typical example of this approach to a practical problem is the *"pseudo-fluid"* simplification. The basic idea is that we assume the suspension to behave like a fluid whose characteristics, density and viscosity, are chosen in order to mimic the actual fluid dynamic interaction experienced by the settling particle.

Pseudo-fluid density can be set equal to the suspension density

$$\rho_{pf} = \phi_s \rho_s + (1 - \phi_s)\rho \tag{9}$$

Pseudo-fluid viscosity, for dilute suspensions, is supplied by the Einstein (1906, 1911) equation

$$\mu_{pf} = \mu(1 + k\phi_s) \tag{10}$$

where, for neutrally buoyant spheres, k is equal to 2.5.

A procedure analogous to the one adopted by Poletto and Joseph (1995) was followed here. They measured single particle settling velocities in a suspension of other particles and through the application of the Stokes' law estimated suspension effective density and viscosity. Differently from this work, suspensions were more concentrated (particle concentrations were varied from 5% to 36%) and only one suspension type was investigated (glass beads in silicon oil).

Similarly, pseudo-fluid viscosities were estimated, through Eq. 1, by assuming the pseudo-fluid density as given by Eq. 9. Figure 4 depicts effective viscosities experienced by a 6 mm glass particle settling into two different suspensions, namely plastic−silicon oil and copper−silicon oil. The effective viscosity of the first suspension, where the solid is nearly neutrally buoyant, confirms the validity of Einstein's theoretical results, whereas for the second suspension a deviation is quite evident.

The neutrally buoyant suspension behavior is well known and documented (for example, Milliken et al. (1989) verified the Einstein equation for $0.17 \leq \lambda \leq 2$ at a suspension volume solid fraction of 5%). When non-neutrally buoyant particles make up the suspension, they settle and, therefore, a fluid backflow takes place, which, in turn, influences the settling velocity of the test sphere. One could think of this as a possible explanation for the deviation in effective viscosity. However, we verified that this effect is always quite negligible. Consider, for example, the worst possible situation depicted in Figure 4: the settling of a 6 mm glass sphere in the 5% copper−silicon oil suspension. The 1 mm copper particles settle at a velocity of 2.7 mm/s, giving rise to a fluid back-flow of 0.15 mm/s. This velocity is much smaller when compared with 12 mm/s, the settling velocity of the glass test sphere. Obviously, the crude pseudo-fluid approach, as described

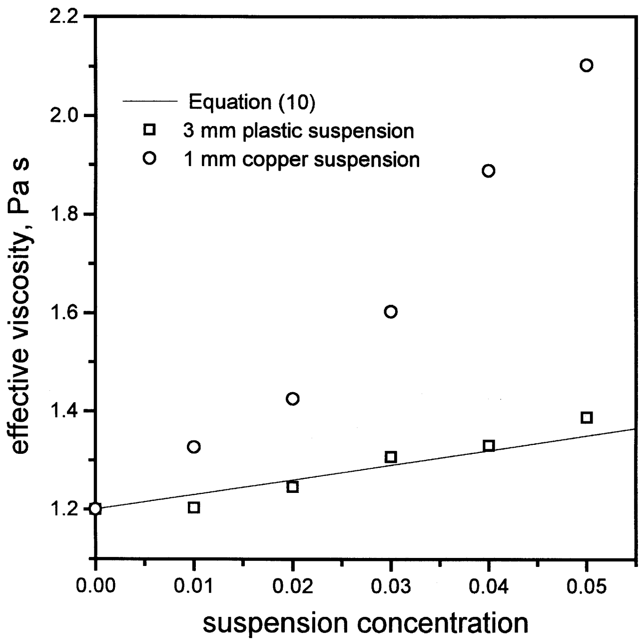

Figure 4. Suspension effective viscosity as experienced by a 6 mm glass sphere in two different suspensions, compared with Einstein's theoretical prediction.

here, cannot completely represent the complex fluid dynamic solid-fluid interaction.

Conclusions

The theoretical predictions of Batchelor have been thoroughly verified for the settling of a single sphere in a suspension of others in a wide range of λ and γ values. When the experimental settling velocities are analyzed in terms of suspension effective viscosity, the Einstein equation is valid, as expected, for neutrally buoyant suspension, but care should be taken when the density of the suspension solid is much different from that of the fluid.

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Notation

d = particle diameter, m
g = gravitational acceleration constant, m/s^2
k = numerical coefficient
n = numerical coefficient
S = numerical coefficient
u = particle settling velocity, m/s
u_t = terminal settling velocity, m/s

Greek letters

γ = relative density ratio
λ = diameter ratio
μ = fluid viscosity, Pa·s
ρ = fluid density, kg/m^3
ρ_p, ρ_s = particle density, kg/m^3
ϕ = solid volume concentration

Subscripts

i = refers to particle type i
j = refers to particle type j
p = refers to a single particle in a suspension of others
pf = refers to pseudo-fluid
s = refers to particle making up the suspension

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